Subject: Leaving Certificate Maths

Teacher: Mr Murphy

Lesson 6: Algebra IV

6.1 Learning Intentions

After this week's lesson you will be able to;

- Solve equations involving surds
- Find the solution set for an inequality of various types (linear, quadratic, rational)
- Prove general inequalities

6.2 Specification

4.3 Inequalities	– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form: • $g(x) \leq k$, $g(x) \geq k$, • $g(x) < k$, $g(x) > k$, where $g(x) = ax + b$ and $a, b, k \in \mathbf{Q}$	– use notation $\mid x \mid$ – select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form: • $g(x) \leq k$, $g(x) \geq k$; • $g(x) < k$, $g(x) > k$, with $g(x) = ax^2+bx+c$ or $g(x) = \frac{ax+b}{cx+d}$ and $a, b, c, d, k \in \mathbf{Q}$, $x \in \mathbf{R}$ • $\mid x - a \mid < b$, $\mid x - a \mid > b$ and combinations of these, with $a, b, \in \mathbf{Q}$, $x \in \mathbf{R}$

6.3 Chief Examiner's Report

The mean mark for questions of this type in 2015 (the last report on mathematics) was between 60% and 70%.

6.4 Surd Equations

In order to solve these equations, we must first understand what a surd is. A surd can be described as a term written using the root symbol that cannot be written as a rational number.

For example:

$\sqrt{4} = 2$

Where two can be written as many different rational numbers e.g. $^2/_1$. Therefore $\sqrt{4}$ is not a surd, however $\sqrt{3}$ is (remember $\sqrt{3}$ is an irrational number also).

A surd equation is an equation including variables and constants but also a surd term. To deal with a surd term we simply carry out the inverse operation. That is, we square both sides of the equation. However, to keep the equation as simple as possible follow these steps.

1) Isolate the surd term (if we have 2 surd terms put one on each side of the =)
2) Then square both sides
3) Solve as linear equation
4) Verify your answer by subbing the x-value into the original equation.

$\sqrt{7x-3}=4$

6.5 Linear Inequalities

An inequality is the opposite to an equation.

Equation: is where both sides of the equation are the exact same magnitude. Gives one value for x.

Inequality: is where one side is greater in magnitude than the other. Gives a range of values for x.

With this there are four symbols in inequalities:

$>$ = LHS is greater than the RHS.
$<$ = LHS is less than the RHS.
$\geq$ = LHS is greater than or equal to RHS.
$\leq$ = LHS is less than or equal to the RHS.

In solving a linear inequality, we can often display the solution set on the number line. In order to do this, we must know what number system (family) the solutions belong to. This is shown in the question using set notation. We covered this back in Week 1 section 1.4.

Solving an inequality:

In solving inequalities, we treat them exactly like equations to obtain a solution similar to $x>3$.

However, we **cannot** just multiply or divide by a negative number **unless** we **flip the inequality** sign also.

Example:

$-x<3$.....Mulitply by -1
$x>-3$.

Compound Inequalitites:

This is where we have two inequalities joined together to make a larger inequality.
To solve these, it is best to split the compound inequality up into the two separate inequalities that make it up.

$$-4 < -\frac{x+2}{3} \leq -3, \qquad where\ x \in \mathbb{R}$$

6.6 Quadratic Inequalities

Quadratic inequalities are generally of the form

$$ax^2 + bx + c\ \_\ 0$$

Where ___ can be one of four symbols: <, >, $\leq$, $\geq$

> =

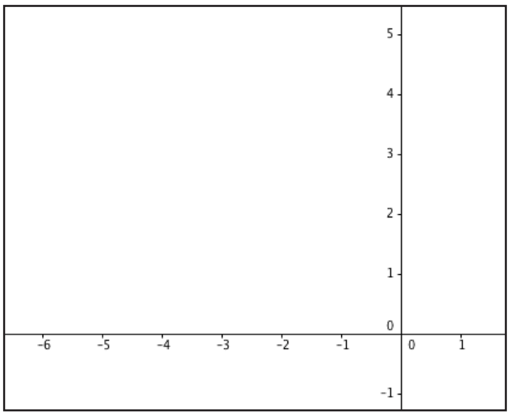
< =

$\geq$ =

$\leq$ =

In solving these inequalities, we treat them as a quadratic equation up until the very end when we graph the solutions on a sketch of the quadratic equation.

$x^2 + x - 12 > 0$



The sketch of the graph is only a sketch, all you need to sketch is the two roots, the y-intercept and the leading coefficient.

Remember the leading coefficient determines whether the graph is a u or n shape.

6.7 Rational Inequalities

These inequalities are usually of the form where there is a fraction whereby both the denominator and numerator are both algebraic expressions. Just like the following example:

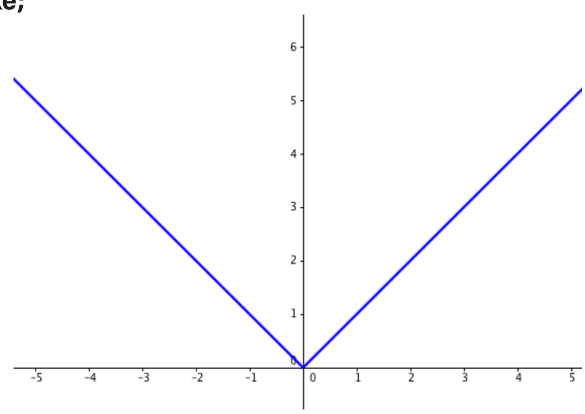
$$\frac{2x-1}{x+2} > 4 \, , x \neq -2$$

6.8 Absolute Value

Absolute value refers to the distance a certain point is on the number line from 0, or the magnitude of the number regardless of sign. Therefore, we always have a positive value after the absolute value is removed.
$|x| = |2| = 2$
Similarly, if x = - 2

$$|x| = |-2| = 2$$

Which on a graph looks like;

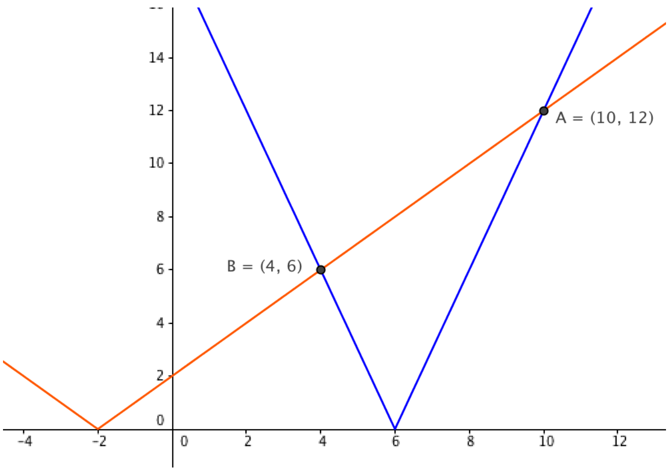


In solving absolute value, we have two options:

1) Using Algebra
2) Using graphs

Using Algebra, we must treat the inequality similar to an equation and solve. To remove the absolute value operation, we must square both sides of the equation and then solve.

$$3|x-6| \leq |x+2|$$



So we are looking for where $3|x-6|$ is less than or equal to $|x+2|$. In other words between what x-values is the blue function under the red.

And it's between 4 and 10 or in mathematical notation

$$4 \leq x \leq 10$$

6.9 Discriminants

A discriminant is what helps us establish a relationship between the initial appearance of a quadratic equation and its roots.

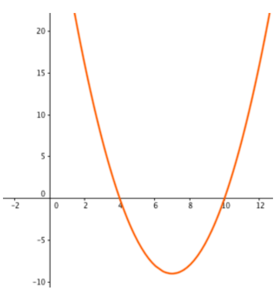
The information comes from the quadratic formula and one part of it in particular.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

But we are only interested in the expression under the square root symbol:
$b^2 - 4ac$
The nature of this expression gives details on both the graph of the expression and its roots.

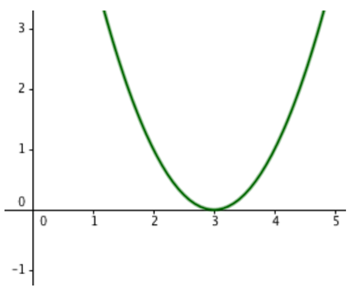
$b^2 - 4ac > 0$

2 distinct real roots



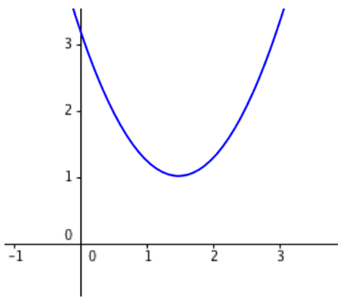
$b^2 - 4ac = 0$

1 repeated real root



$b^2 - 4ac < 0$

No real roots



We can be told how many real roots an equation has and use the idea of discriminants to identify unknown coefficients or constants

Find the value of q if this equation has no real roots

$$x^2 + qx + q = 0$$

6.10 Proofs of Inequalities

We can be asked to prove certain inequalities are true

To do this:

1) Rearrange the inequality so it reads ... ≥ 0
2) Look for a reason as to why the L.H.S. is positive.

The reason we do this is because we can say that:

Any $(real\ number)^2$ is positive

Look at the sample in the video and fill in the example below

$$Prove\ a^2 + b^2 - 8a \geq -16, for\ all\ a, b\ \&\ c \in \mathbb{R}$$

6.11 Recap of the Learning Intentions

After this week's lesson you will be able to;

- Solve equations involving surds
- Find the solution set for an inequality of various types (linear, quadratic, rational)
- Prove general inequalities

6.12 Homework Task

1) $5(2x-1) > 2(x-1)+5,\ x \in \mathbb{N}$....don't forget to graph solutions on number line.

2) $2(3-2x) > 2(3-x)-3+x,\ x \in \mathbb{R}$

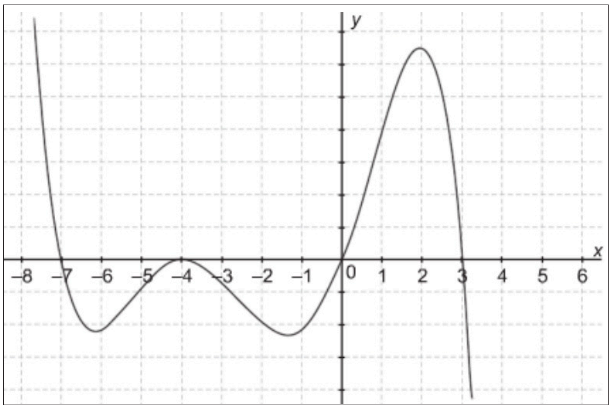
3) $-72 \geq -5x^2 + 54x$

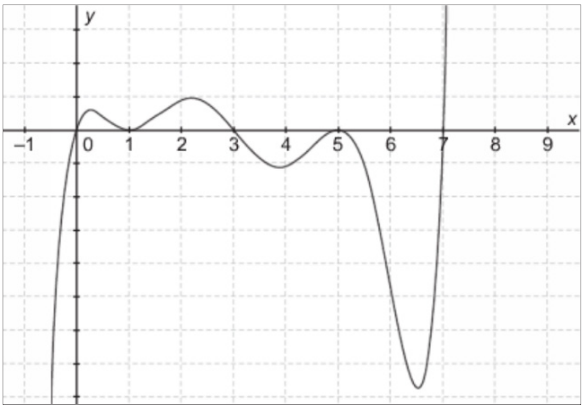
4) $\frac{3x-7}{x-4} < 2,\ x \neq 4$

5) Prove that for all values of $\mathbb{R}$ $(a-2)x^2 + 2x - a = 0$ has real roots

6.13 Solutions to 5.8

Find the roots and thus the polynomial of the below graphs





Roots: -7, -4, -4, 0, 3

Factors: $(x+7)(x+4)^2(x)(x-3)$

y = $x^5+12x^4+27x^3-104x^2-336x$

Roots: 0, 1, 1, 3, 5, 5, 7

Factors: $(x)(x-1)^2(x-3)(x-5)^2(x-7)$

y = $x^7-22x^6+187x^5-772x^4+1591x^3-1510x^2+525x$

Solve the following simultaneous equation:

$x^2+y^2-13=0$
$5x-y=-13$

$-y=-13-5x$
$y=13+5x$

$x^2+(13+5x)^2-13=0$
$x^2+169+130x+25x^2-13=0$
$26x^2+130x+156=0$
$x=-3 \text{ or } x=-2$

$y=13+5x$
$y=13+5(-3)$ $\quad$ $y=13+5(-2)$
$y=13-15$ $\quad$ $y=13-10$
$y=-2$ $\quad$ $y=3$

Final Solution: Points of Intersection are (-3,-2) and (-2,-3)